$(T, \eta, \overline{(-)})$ Kleisli Triple

- $T : \mathrm{obj}\, \mathcal{C} \longrightarrow \mathrm{obj}\, \mathcal{C}$
- $\forall a \in \mathcal{C} .\ \eta_a : a \to Ta \in \mathcal{C}$
- $\forall f : a \to Tb \in \mathcal{C}$ $\quad \bar{f} : Ta \longrightarrow Tb \in \mathcal{C}$

$$\begin{tikzcd} Ta \arrow[dr, dashed, "\overline{\eta_a}"] & \\ a \arrow[r, "\eta_a"'] & Ta \end{tikzcd}$$

① $\overline{\eta_a} = 1_{Ta}$

② $\bar{f} \cdot \eta_a = f$

③ $\overline{\bar{g} \cdot f} = \bar{g} \cdot \bar{f}$

— — — — — —

$(b \xrightarrow{g} Tc) \circ (a \xrightarrow{f} Tb) = \bar{g} \cdot f$

$T : \mathrm{obj}\, \mathcal{C} \longrightarrow \mathrm{obj}\, \mathcal{C}$ $\qquad a \xrightarrow{f} Tb \xrightarrow{\bar{g}} Tc$

Monad

(T, η, μ)

$$
\begin{tikzcd}
 & T^3 \arrow[dl, Rightarrow, "\mu T"'] \arrow[dr, Rightarrow, "T\mu"] & \\
T^2 \arrow[dr, Rightarrow, "\mu"'] & & T^2 \arrow[dl, Rightarrow, "\mu"] \\
 & T &
\end{tikzcd}
$$

$\eta_a : a \to Ta$

$\mu_a : T^2a \to Ta$

$$
\begin{tikzcd}
T \arrow[r, Rightarrow, "T\eta"] \arrow[dr, equal] & T^2 \arrow[d, Rightarrow, "\mu"] & T \arrow[l, Rightarrow, "\eta T"'] \arrow[dl, equal] \\
 & T &
\end{tikzcd}
$$

- $\mu_a \circ \mu_{Ta} = \mu_a \cdot T\mu_a$
- $\mu_a \cdot \eta_{Ta} = 1_{Ta} = \mu_a \cdot T\eta_a$

T

From this data, construct a Kleisli Triple

$(T, \eta, \overline{(-)})$

$$
f : a \to Tb \quad \longmapsto \quad
\begin{tikzcd}
Ta \arrow[r, "Tf"] \arrow[dr, "\bar{f}"'] & T^2b \arrow[d, "\mu_b"] \\
 & Tb
\end{tikzcd}
$$

$(T, \eta, \mu) \rightsquigarrow (T, \eta, \overline{(-)})$

Interpretation of equation $\bar{f} \cdot \eta_a = f$

$\eta_a : a \longrightarrow Ta$

$f : a \longrightarrow Tb$

$\bar{f} : Ta \longrightarrow Tb$

$\eta_a ; \bar{f} = f$

takes a value of type "a", and gives the program of type "Ta" (which returns the same value) ~~of~~

f transform a value of type "a" to a programme of type "b"