William Lawvere on Myles Tierney*

October 26, 2017

Dear Friends and Colleagues,

I am deeply sad about the loss of Myles, my friend and a pillar of the community surrounding Eilenberg and Mac Lane. I highly respected him as a creative collaborator. Whenever we met at meetings, or spoke on the phone, even if we had not seen each other for a long time, we knew each others way of thinking.

Thanks to Andre Joyal for his obituary and the wealth of information about the work of Myles Tierney. I elaborate below on our collaboration.

Myles and I had independently recognized the need for an axiomatic theory of sheaves and related matters, and at a gathering in Albrecht Dolds house near Heidelberg, we agreed to collaborate on the construction of such a theory during our upcoming stay at Dalhousie University. (Myles had agreed to join our group of researchers for a year.) Already in the first days of our seminar Myles made important advances, such as formulating the axioms of exactness which we knew would have to be theorems of a correct theory of topos.

He emphasized in general that Grothendieck had made the category (rather than the space) the central aspect that we should explicitly axiomatize. We had a pre-publication copy of SGA4 that we consulted frequently. In that work there was a significant advance over previous formulations of the sheaf condition: In a pre-sheaf topos, a covering specified by a Grothendieck topology, was no longer an infinite family of subobjects, but a single subobject R, (which of course might be imagined to arise as the union of a family); this made possible the formulation of notions in finitary terms, indeed in terms of a single operator whose properties Myles made precise. (Some people object to calling such operators topologies; the same objection applies to Grothendiecks use of the term topologies for his equivalent notion, which is of course also not literally a topology in the classical sense. Later we referred to such an operator as a localness operator, as a modal operator it is locally the

*Here is the obituary that William F. Lawvere wrote after death of his long-term collaborator Myles Tierney on category theory mailing list. All credit is due to Lawvere. This document is only meant to store Lawvere's well-written and touching obituary since I could not find a link to it on category mailing list website and I copied it form my email inbox. Nothing has been added or retracted from Original email sent by Lawvere.

case that, or as a Tierney closure operator which - as I pointed out to Kuratowski on his visit to Dalhousie - is not a Kuratowski closure operator since it preserves intersections, rather than unions. Similar operators arise in other parts of mathematics where they are sometimes called 'nuclei'.)

Another unique feature of SGA4 is that it contains no definition of topos; indeed every rigorous mention is of U-topos. This parameter U was essentially a model of set theory, and previous work on the category of sets showed clearly that for mathematical purposes, the use of composition of mappings is more effective than towers of membership. Thus Myles and I decided to replace U itself by an arbitrary topos (in our determination of that term), and indeed a general U-topos E could now be seen as structured by a morphism $E \to U$. This provided a suitable codomain for the 2-functor from internal U-sites to U-toposes, whose image consisted of those E which contain a bound in U. That, our fundamental preliminary goal, was proved in his thesis by the remarkable student of Myles, the late Radu Diaconescu.

We also had available a preliminary version of Monique Hakims thesis applying Grothendieck's notion of classifier topos to parameterized complex analysis. This notion can be seen as a key step of Model Theory, except that the use of conjunction and disjunction and existential quantification is replaced by Grothendieck's direct recognition of which classes of structures are defined in terms of finite limits and small colimits. Of course, a logician will expect or want primitive predicates and basic axioms to present such notions of structure, and to facilitate such recognition. Our initial work to make explicit such a notion of theory-presentation was carried out by several people. It became evident that such a construction does not work for general elementary theories, because the negative operations of universal quantification and implication are not preserved by the geometrical morphisms of toposes, even though these operations are well-defined and exist within each particular topos. (Restricting to open geometric morphisms or to positive presentations can partly circumvent this limitation.) The positive presentations need to use the classical idea of sequent, rather than the mere specification of a class of formulas that aim to be true.

The classifier toposes are useful for mathematical questions other than logical ones, for example in combinatorial topology, as first pointed out in detail by Andre Joyal. The relation between combinatorial schemes and the spaces they generate is an adjoint functor. Specifically, any structure carried by a unit interval or by other basic space in a topos of spaces gives rise to the adjoint between the topos of spaces itself and the classifying topos for such structures, for example, distributive lattice, Boolean algebra, total ordering. Myles foresaw such clarifying applications, and in many other ways his knowledge of topology supplemented my own background.

A useful construction made explicit by Myles is still not widely recognized: the category of co-algebras for a given left-exact comonad in a given topos, is indeed another topos. This can be seen as an essential step in the construction of a pre-sheaf topos.

A significant advance in the world of logic was the result of Diaconescu, showing that the axiom of choice implies Boolean logic. His professor Myles Tierney had proved the independence of the continuum hypothesis, making essential use of the notion of sheaf.

(LNM 274 [1]). It was in preparing his talks for our seminar 1969-1970 that Myles formulated most of the logic needed for that result. Sabah Fakir who took part in these seminars sent notes each week to Jean Benabou who also gave a seminar simultaneously. Anders Kock was a very active participant, whose later Aarhus seminar with Gavin Wraith also contributed to the rapid dissemination of the results and the general point of view.

After I had presented our results at the 1970 ICM, we organized an international meeting in January 1971; Myles returned from Rutgers to Halifax to explain the continuum hypothesis to the 70 participants.

Myles and I were gratified in the ensuing months and years to see that our general point of view was taken up by many mathematicians and logicians. Further applications, for example the role of Boolean sheaves in algebraic geometry, still await detailed publication. Several works by Michael Barr intiated such applications.

In 1974 Myles Tierney and Alex Heller organized a meeting in New York City in honor of Samuel Eilenbergs 60th birthday; they edited the proceedings as Algebra, Topology, and Category Theory [2]. That book contains an influential article by Myles on the construction of classifier toposes for internal sites. Myles Tierney, Fred Linton, Jon Beck and I were roughly the same age when we started our studies at Columbia University; Mike Barr, Peter Freyd, John Gray, and Barry Mitchell were senior to us, and collaborated with us too. Sammy had attracted a group, all of whom continued to contribute to category theory.

In the ensuing years Myles organized a regular seminar in New York on weekends. It was attended by categorists from several states. I attended the seminar frequently from Buffalo, and I took along several students including Kimmo Rosenthal and Phil Mulry. Thanks to the generosity of Myles and Hanne, everybody was invited to sleep on the floor of their loft.

When I last spoke with Myles on the phone just a few months ago he explained to me the specific topological intuition underlying the notion of Kan complex.

I know that Myles would have joined me in welcoming any future mathematics on the horizon for which our work might have provided a stepping stone.

Bill Lawvere

References:

[1] Toposes, Algebraic Geometry and Logic. LNM 274, Springer 1972

[2] Algebra, Topology, and Category Theory, Academic Press, 1976