Mathematics & The Mind

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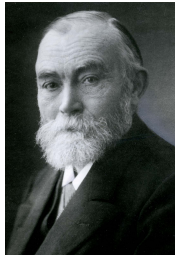
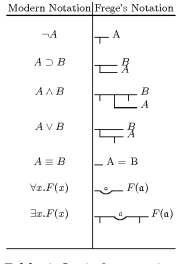
Frege

- Philosophy of Mathematics, after Frege, has been dominated by the methods of logic. It is by now a fully established part of analytic philosophy.
- It has been very successful in certain respects (e.g. foundation of mathematics), but has not been much helpful in other respects (e.g. epistemology of mathematics).
- This kind of research in the philosophy of mathematics has begun to stagnate.
- The philosophy of mathematics has to attend to the kinds of value judgments that govern everyday mathematical practice, and provide a more realistic description of the goals and purpose of mathematical activity.



Gottlob Frege (1848-1925)



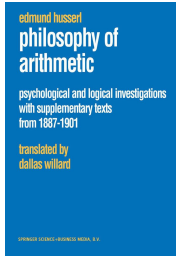
Frege's Logic

Husserl's Philosophy of Arithmetic (I)

In "Philosophie der Arithmetik" (1981), Edmund Husserl

- advocates grounding mathematical knowledge on more secure and less erroneous foundations through a sequence of *psychological* and logical investigations
- aimed to phenomenologically understand the *subjective origins* (e.g. intentionality) of the fundamental concepts of set theory and finite cardinal arithmetic.
- aimed at providing a psychological analysis of the concepts multiplicity, unity, and number, insofar as they are given to use *authentically* and not through indirect *symbolizations*.

Edmund Husserl (1859-1938)



Philosophy of Arithmetic

Husserl's Philosophy of Arithmetic (II)

INTRODUCTION

There are various concepts of *number*. This is already indicated by the different words for numbers that show up in the language of ordinary life. These are usually listed by grammarians under the following headings: The whole numbers [*Anzahlen*] or cardinal numbers (*numeralia cardinalia*), the ordinal numbers (*n. Ordinalia*), the type numbers (*n. specialia*), the numbers of repetition (*n. iterativa*), the multiplicative numbers (*n. multiplicativa*), and the fractional numbers (*n. partitiva*). That the whole number comes first in this sequence is, as with the other characteristic names which it also bears – the "basic" or "cardinal" number – not founded on mere convention. Linguistically it assumes a privileged position because all of the remaining number terms proceed only through slight modifications from the words for the whole numbers. (E.g., two, second, of two kinds, double, twice, half).[1] The latter are thus genuine terms for basic numbers. In this way language guides us to the thought that the corresponding *concepts* also may all stand in an analogous relation of dependence to those of the whole numbers, and may represent certain

Husserl's Philosophy of Arithmetic (III)

In the spirit of phenomenology, Husserl employs dynamic terms to characterize the way certain mathematical concepts arise and the role they play in thought.
For example:

- a *collective combination* arises from "focusing attention" on the relationship between objects in a group, and "noticing" that they share something in common.
- a *multiplicity* then arises from "seeing" the objects as units, and "disregarding" their individual nature.
- a *number* arises from the process of "thinking of" a multiplicity as an answer to the question, "how many?"

Husserl's Philosophy of Arithmetic (III)

According to Husserl, *abstract concepts* arise from a process of "disregarding" certain aspects of a particular perception, "directing" attention at certain others, "isolating" those aspects, and "intending" the concept to be the *result of those acts*.

> *To disregard or abstract from something means merely to give it no special notice. The satisfaction of the requirement wholly to abstract from the peculiarities of the contents thus absolutely does not have the effect of making those contents, and there with their combination, disappear from our consciousness. The grasp of the contents, and the collection of them, is of course the precondition of the abstraction. But in that abstraction the isolating interest is not directed upon the contents, but rather exclusively upon their linkage in thought – and that linkage is all that is intended.*

Frege vs. Husserl

- The work of Husserl in philosophy of mathematics is largely neglected until today by philosophers of mathematics.
- This was in large part due to Frege's ruthless and adversarial review of Husserl's book.
- In his Grundlagen der Arithmetik of 1884, Frege railed against what he perceived as psychologism in Husserl's philosophy of mathematics.
- In fact, Husserl himself had written extensively against psychologism.
- Nonetheless, Frege is unsympathetic to Husserl's views. For instance this is what Frege thinks of Husserl's account of abstraction:

> *We attend less to a property and it disappears. By making one characteristic after another disappear, we get more and more abstract concepts. . . Inattention is a most efficacious logical faculty; presumably this accounts for the absentmindedness of professors.*

The Fallout of Frege-Husserl controversy

- The origin of continental-analytic split
- Frege's anti-psychologism was fully embraced by the analytic camp.
- Husserl clearly felt the need to respond to Frege's invective and distinguish his project from brute psychology and to renounce psychologism (e.g. the Prolegomena to the first volume of his Logische Untersuchungen).
- Husserl's later transcendental idealism reinforced the distinction between his philosophical program and psychology by characterizing the former as a determination of the essential capacities of an *idealized mind*, rather than the determination of our own mental capacities.
- Frege never explained whence the concept of number he present derives its normative force.

Humans and mathematics

Humans

- can think about *abstract* things.
  - abstract things are non-spatial, non-temporal things like numbers , emotions, etc.
- possess cognitive abilities that extend far beyond their *immediate sensory experiences*.
- find themselves in a complex world with limited cognitive resources. These restraints on cognitive resources are a defining component of mathematical activity (e.g. abstraction, axiomatic systems).
- The nature of these cognitive resources are relevant in finding out the nature of mathematics itself.

Frege's Influence: Cognition As Symbol Manipulation

Concerning the contribution of logic to the early days of cognitive science, we see two roles of logic:

- *Descriptive*: human beings were taken to be fundamentally logical, or rational.
- *Normative*: logic was taken to define rational behavior and thus to provide a starting point for the artificial reproduction of intelligence.

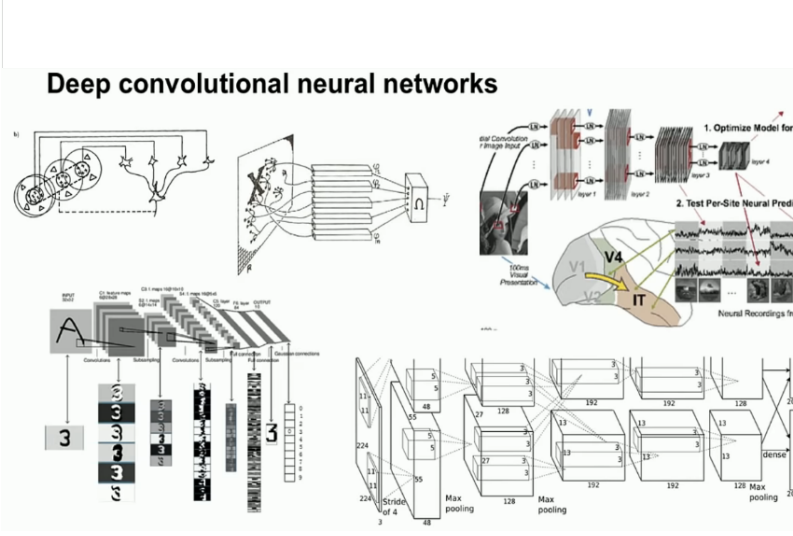
Examples are

- "A logical calculus of the ideas immanent in nervous activity" by McCulloch and Pitts (1943).
- Abstract Theory of Computation by Alan Turing (1950).
- Church-Turing thesis
- Turing-complete Programming Languages (starting with λ-calculus)

Alan Turing

Cognition as Pattern Recognition

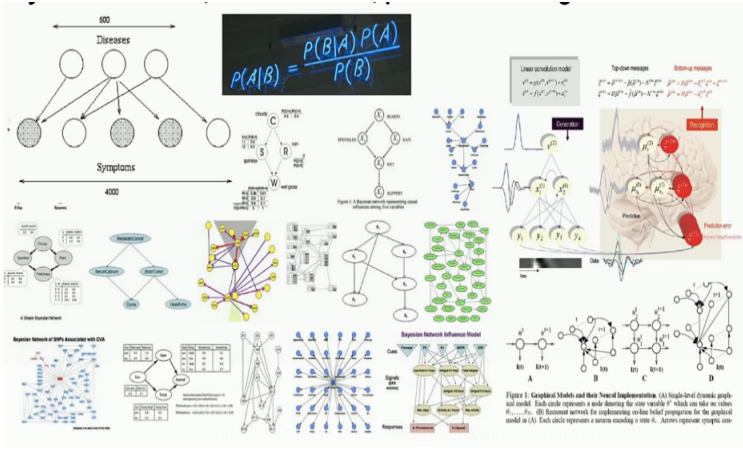
- Deep convolutional neural networks (multi-layer perceptrons)
- From single neuron to multi-layer hierarchy
- Using large date sets
- Remarkable success in speech/vision recognition
- The direction of arrows: sensory input → the decision output, or data → pattern.



Source: Tennebaum, BMM 2018

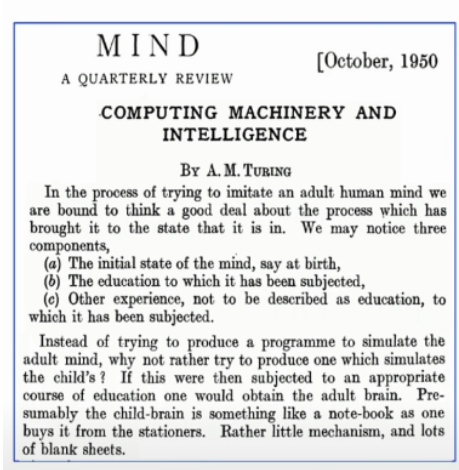
Cognition as Prediction Engine

- Bayesian network as engine for probabilistic inference
- Sometimes for causal inference
- Modeling top-down expectation of how the world around us works
- The direction of arrows: causes → effects, e.g. disease → symptoms



Source: Tennebaum, BMM 2018

Memory as Infinite Tape



MIND

A QUARTERLY REVIEW

[October, 1950

COMPUTING MACHINERY AND INTELLIGENCE

BY A. M. TURING

In the process of trying to imitate an adult human mind we are bound to think a good deal about the process which has brought it to the state that it is in. We may notice three components,

(*a*) The initial state of the mind, say at birth,

(*b*) The education to which it has been subjected,

(*c*) Other experience, not to be described as education, to which it has been subjected.

Instead of trying to produce a programme to simulate the adult mind, why not rather try to produce one which simulates the child's? If this were then subjected to an appropriate course of education one would obtain the adult brain. Presumably the child-brain is something like a note-book as one buys it from the stationers. Rather little mechanism, and lots of blank sheets.

"Presumably the child-brain is something like a note-book as one buys it from the stationers. Rather little mechanism, and lots of blank sheets."

Critique of the neurosymbolic paradigm

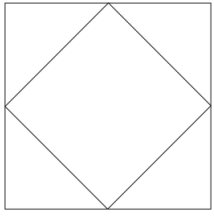
- The most common neurosymbolic computational claims about cognition is a *psychological version of the Church-Turing Thesis*: the human mind can only deal with computable problems. In other words, cognitive tasks comprise computable functions.
- Cognitive systems are biological/physical systems, they perform tasks under computational resource constraints.
- The "functions" computed by cognitive systems need to be computable in realistic time and with the use of a realistic amount of memory.

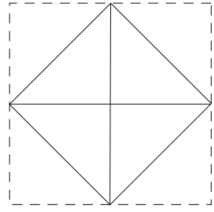
Critique of The Prediction Engine Paradigm

- Humans do not distinguish numbers, say 7 and 8, probabilistically, they distinguish them as concepts (categorical distinctness).
- The arithmetic depends on the grasp of numbers as concepts, not on the number sense.

Generalization

If “the inner square” is the square whose vertices are midpoints of the sides of a square (“the outer square”), the area of the inner square is half the area of the outer square.

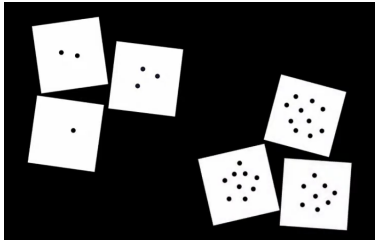




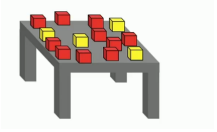
Numbers: Back to Husserl

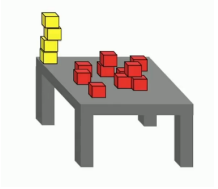
- Number has several meanings depending on different contexts/use
  - every-day use: ticket number, telephone number, house number, a number of items, number of letters of a word, etc.
  - technical use: cardinal number, ordinal number, transfinite number, irrational number, complex number, hyper-real number, etc.
- The cognitive literature is filled with a plethora of confusing terminology (a term with two distinct meanings, and multiple terms with the same sense).
  - numeral
  - numerosity
  - numerousness
  - quantity
  - quantical
  - exact
  - approximate
  - symbolic
  - ...

The Sense of Numbers



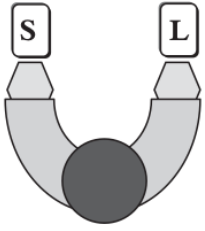
Subitizing

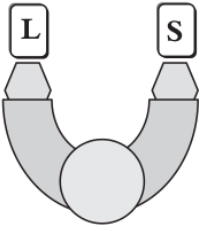




Block tables

- 'Is the presented number *Smaller* or *Larger* than 65?'
- SL subjects responded faster (and with fewer errors) than LS subjects

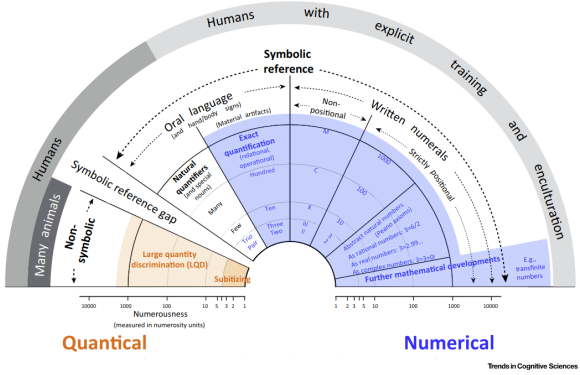




What is the origin of numbers? (I)

- Naturalistic account: biologically evolved capacity for numbers specifically
- Linguistic account: Numbers as linguistic devices like quantifiers
- Behaviourist account
- Anthropological account: numbers as a technological tool (product) of historical (epochal) evolution of civilizations/cultures

What is the origin of numbers? (II)



Rafael E. Núñez, Is There Really an Evolved Capacity for Number?

The Anthropological Account

- Material tools (including organs, e.g. Pentadactylism)
- Grammatical/Lexical tools
- Writing tools
- Cultural practices
- Symbolic tools and symbolic reference
- Cognitive offloading
- Schooling/learning rather than individual acquisition

Number, in particular is language-mediated, conventionalized, exact symbolic quantification.

Man himself produces man

"The ethical programme of the present came into view for a moment when Marx and the Young Hegelians articulated the theory that man himself produces man. The true meaning of this statement was immediately obscured, however, by another chatter that presented work as the only essential human act. But if man genuinely produces man, it is precisely not through work and its concrete results, not even the 'work on oneself' so widely praised in recent times , let alone through the alternatively invoked phenomena of 'interaction' or 'communication'; it is through life in forms of practice. Practice is defined here as any operation that provides or improves the actor's qualification for the next performance of the same operation, whether it is declared as practice or not."

Peter Sloterdijk: Du mußt dein Leben ändern

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